Fault Tolerant Communication Topologies for Wireless Ad Hoc Networks

Bernd Thallner*
Technische Universität Wien, Embedded Computing Systems Group E182/2
Treitlstrasse 3, 1040 Vienna, Austria, thallner@ecs.tuwien.ac.at

Abstract

We present a construction of a fixed node-degree overlay network that facilitates efficient fault-tolerant multi-hop communication in large-scale distributed systems: Given a weighted graph where the weight of an edge represents the cost of a connection, the constructed subgraph is \( \Delta \)-regular, \( \Delta \)-node connected, ensures failure locality, and has low total weight. Moreover, there is a distributed algorithm for constructing this subgraph, which adapts to dynamic changes of the environment, is guaranteed to converge, and exhibits good average case performance as well. As a by-product, our construction builds a hierarchy of clusters that reflects the node density in the network, with guaranteed and localized fault-tolerant communication between any pair of cluster members. It is hence well suited for both establishing fault-tolerant communication topologies and clustering in wireless sensor networks, and for constructing robust overlay graphs in peer-to-peer systems.

Keywords: Wireless sensor networks, fault-tolerant communication, clustering, overlay networks, redundant paths, distributed algorithms, failure locality

1 Introduction

Wireless sensor networks, mobile ad hoc networks and peer-to-peer systems are examples of distributed applications that cannot reasonably be modeled as fully connected systems. The huge number of participants and/or resource limitations do not allow a node to communicate with—or even to know—every other node in the system. Rather, every node can communicate directly with a small number of neighbors only. For example, typical CDMA-based ad hoc wireless networks [12] need some dedicated hardware (e.g. a correlator) on both sender and receiver side of every point-to-point connection. Consequently, given the limited resource and power budget of wireless devices, only a relatively small number (say, \( 2 \ldots 10 \)) of connections to near-by nodes can be maintained simultaneously. Note that such connections are bidirectional and typically long-living (i.e., used for multiple messages), since spread spectrum communication involves an expensive communication setup phase due to synchronization etc.

In addition, an individual cost figure (weight) is usually associated with a connection. It measures how expensive or difficult it is to communicate with a specific peer. Distance, required transmission power, interference level or any combination of such quantities are legitimate cost figures. Note that connection weights need not satisfy the triangle inequality. In addition, connection weights are usually time-dependent: The required transmit power for communication with a certain node in a CDMA wireless system, for example, does not only depend upon the distance but also upon the instantaneous level of the internal (thermal) noise, multiuser interference, signal attenuation, multi-path fading, and many more. Likewise, a connection to a receiver that temporarily suffers from low battery or heavy processor load may be considered more costly than usual.

From a system’s point of view, the actual connections between direct neighbors form a (sparse) overlay graph. It must be constructed in a way that allows simulating a fully connected network via multi-hop communication: The overlay graph must be spanning and should have low total weight for communication/resource efficiency. In real systems, the problem of constructing/maintaining such an overlay graph is further exacerbated by the fact that link performance can degrade, that both links and nodes can crash and recover, and nodes can move. Note that moving nodes correspond to changing edge weights. In order to guarantee unimpaired communication under such circumstances, the overlay network must be robust: It should (1) provide multiple disjoint paths between any pair of nodes to guarantee timely delivery of every single message, despite failures, and (2) adapt quickly to more persistent changes in the environment. Suitable overlay graphs must hence be more general than spanning trees and should be maintained in a cost-optimal and self-healing way: If connections or nodes become expensive or go down for some time, the overlay graph must be adapted in order not to use them further.

Accomplishments: We present a method for constructing a \( \Delta \)-regular overlay graph for efficient fault-tolerant multi-

*This research is part of the W2F-project, which has been supported by the Austrian START program Y41-MAT, see http://www.auto.tuwien.ac.at/Projects/W2F/
hop communication in large-scale distributed systems. It is based on recursively forming groups consisting of $\Delta$ nodes that are treated like single nodes subsequently, thereby building a hierarchy of groups represented by a $\Delta$-ary tree that actually reflects the node density. The constructed overlay graph is $\Delta$ node-connected, which is optimal, has low total weight, and inherently ensures failure-locality: Even excessively many failures in some part of the system do not impair fault-tolerant communication in other parts.

Moreover, there is a fully distributed algorithm\(^1\) for constructing the topology. The algorithm automatically adapts to a changing environment (i.e., varying connection weights, node crashes & joins) and is guaranteed to converge when the environment is stable for sufficiently long. Moreover, by choosing different propose modules (which suggest proposals for new groups to the algorithm), the minimality of the overlay graph’s total weight can be traded for message complexity. Our worst case performance analysis reveals a message complexity from $O(n)$ to $O(n^{\Delta+1})$, depending upon the propose module used. Moreover, some simulation experiments indicate that the average complexity of our algorithm is linear in $n$. We are hence convinced that our approach is indeed well suited for establishing fault-tolerant communication topologies and clustering in large-scale wireless sensor networks, as well as for constructing robust overlay graphs in peer-to-peer networks.

Related Work: Most work in this area either deals with nodes that are position-aware [16] or is based on the unit disk graph (UDG) model [3] where a link between two nodes exists iff their Euclidean distance is at most one. On top of this model, a number of solutions [1, 2, 4–7, 10, 14] for constructing spanners have been proposed. A spanner is an overlay graph, which guarantees that any two nodes have a distance of at most a constant factor times the minimum distance in the UDG graph. Distributed algorithms for constructing spanners have been presented [1, 6, 10, 14] as well. Still, none of those solutions ensures redundant paths for fault-tolerant communication and maintains a fixed and small node degree. In [9] an algorithm for transforming an arbitrary spanner into a fault-tolerant spanner is presented; it is not distributed, however. Finally, there are probabilistic algorithms for the related problem of constructing fault-tolerant overlay networks for peer-to-peer systems [11].

The remainder of our paper is organized as follows: In Section 2 we define our basic notation. Section 3 introduces our group construction method and proves the properties of the resulting overlay graph. A summary of our accomplishments and some directions of further work in Section 4 concludes the paper.

\(^1\)Lack of space does not allow us to include the algorithm and the performance analysis in this paper, see [13] for details.

2 Definitions

We consider simple undirected weighted graphs $G = (\Pi, \Lambda)$ consisting of a set of $n$ nodes $\Pi = \{1, ..., n\}$ and a set of weighted edges $\Lambda \subseteq \Pi \times \Pi \times \mathbb{R}$ with $(x, x, w) \notin \Lambda$. A graph is called $\kappa$-node-connected if the removal of any subset of $\kappa - 1$ nodes leaves the graph connected while there exists a subset of $\kappa$ nodes whose removal disconnects the graph. A graph is said to be regular of degree $r$ if all nodes have degree $r$.

Our network is modeled as a simple undirected weighted graph $G$, called communication graph, which is assumed to be fully connected, in the sense that $\omega < \infty$ for any edge $(x, y, \omega)$, $x \in \Pi$ and $x \neq y \in \Pi$. Note carefully that this assumption does not mean that any node must actually communicate with every other node.

Our method constructs a low weight overlay graph $G' = (\Pi, C)$ with $C \subseteq \Lambda$ that is $\Delta$-regular and $\Delta$-node-connected if $n \cdot \Delta$ is even or $\Delta - 1$-node-connected if odd. In order to easily distinguish the communication graph $G$ and the overlay graph $G'$, the edges of the latter will be called connections. Note that the set of alive nodes and the weight of the edges in $G'$ may be time-variant. For convergence of the overlay graph $G'$, the communication graph $G$ must be stable sufficiently long, however.

3 Topology Construction Method

In this section, we provide an overview of our clustering scheme which induces an overlay graph $G'$ with the desired properties. The idea behind the topology construction is to build groups of nodes that appear like single nodes such that they can be treated like those subsequently.

Figure 1 shows an example, where (c) is the fully connected communication graph $G$, (b) depicts the constructed overlay graph $G'$ for $\Delta = 3$, and (a) provides the tree representation corresponding to the constructed topology. From (b) it is apparent that nodes $(1, 2, 3), (5, 6, 7)$ and $(8, 9, 10)$ are combined into groups with id $A$, $B$, and $D$, respectively. Such a group is formed if all members agree upon the fact that the sum of the weights of their internal connections (e.g., $5 - 6$, $5 - 7$, $6 - 7$) is minimal over all alternative group constructions. Each of the $\Delta$ members of a group is connected to all of the $\Delta - 1$ other members (internal connections) and has exactly one connection left (external connection). Since there are $\Delta$ members in a group, any group has—a node—$\Delta$ external connections left, which are available in higher level groups. For example, group $C$ again consists of three members: A single node 4 and two groups $A$ and $B$, which are connected via their external connections. Again, all members of $C$ agree upon minimality of the sum of their internal connections’ weights. Group $R$ finishes the topology and consists of group $C$ and $D$; note...
that it is necessarily incomplete in that it has less than $\Delta$ members. Figure 1.(a) reveals that the resulting group structure is a $\Delta$-ary tree (except for the root, which corresponds to the group $R$ and may hence have < $\Delta$ children). The edges of the tree represent the membership relation among the groups.

We now give a formal description of our topology: Groups consist of an identifier $g_i$ and a set of $\text{members}(g_i)$. The set of all group identifiers is $\mathcal{G}$. The set of $\text{members}(g_i)$ consists of exactly $\Delta$ nodes and groups: $\text{members}(g_i) \subseteq (\mathcal{G} \cup \Pi)$. A node or group can only be member of a single group: $\forall g_a, g_b \in \mathcal{G}, g_a \neq g_b : x \in \text{members}(g_a) \land y \in \text{members}(g_b) \Rightarrow x \neq y$. For every $g_i \in \mathcal{G}$ we define the nodes of a group as $\text{nodes}(g_i) = \bigcup_{k=0}^{\infty} \text{members}^k(g_i) \cap \Pi$ where $\text{members}^0(g_i) = \text{members}(g_i)$ and $\text{members}^1(g_i) = \bigcup_{k \in \text{members}^{-1}(g_i), k \in \mathcal{G}} \text{members}(k)$. For every node $p \in \Pi$ we define $\text{members}(p) = \{p\}$ and $\text{nodes}(p) = \{p\}$.

A connection $(p, q, w) \in C$ from node $p$ to node $q$ with weight $w$ is a group $g_i$ internal connection if $p \in \text{nodes}(g_a)$ and $q \in \text{nodes}(g_b)$ with $g_a, g_b \in \text{members}(g_i)$. If there is a connection $(p, q, w) \in C$ between node $p \in \text{nodes}(g_a)$ and node $q \in \text{nodes}(g_b)$ we call the groups $g_a, g_b$ connected. The members of a group are fully connected: $\forall g_a, g_b \in \text{members}(g_i), g_a \neq g_b, p \in \text{nodes}(g_a), q \in \text{nodes}(g_b) \Rightarrow \exists (p, q, w) \in C$.

Hence, every group member has $\Delta - 1$ connections to other group members and therefore exactly one connection left. Since there are $\Delta$ members in a group, the group has—one node—$\Delta$ connections left. We call the $\Delta$ nodes of a group $g_i$ with one connection left the terminal nodes $T_{g_i} \subseteq \Pi$ of a group. By convention, we define that $T_p = p$ for a single node $p \in \Pi$ and say that every node has $\Delta$-node-disjoint paths to itself. The terminal node of a group with the smallest id is called leader of the group.

**Definition 1.** The weight of a group $\omega(g_i)$ is a tuple $(A, B)$. Value $A$ is the maximum of the sum of all group $g_i$ internal connection weights and the maximum of all group members’ weights plus an arbitrary small constant $c$. Value $B$ is the ID of the group leader. A group $g_i$ has smaller weight than $g_j$, formally $\omega(g_i) \prec \omega(g_j)$, if $A_i$ is smaller than $A_j$ or, if equal, the leader ID $B_i$ is lower than $B_j$.

Note that this definition implies that the weight of a parent group is always higher than the weight of any of its members. This property is required for ensuring that the minimum admissible overlay graph introduced in Definition 3 is well defined, and that our distributed algorithm eventually converges.

**Definition 2.** An overlay graph $G'$ is called admissible if its
corresponding topology has a single root group $g_{\text{root}}$ with $\text{members}(g_{\text{root}}) \subseteq \mathbb{G}$ and $2 \leq |\text{members}(g_{\text{root}})| \leq \Delta$. The root group is constructed as a ring connecting $\left[\frac{\Delta}{2}\right] + \left\lceil\frac{\Delta}{2}\right\rceil$ terminal nodes of every subgroup with its neighboring subgroups.

Note that single-node members in the root group are forbidden in an admissible overlay graph. Moreover, we actually allow overlay graphs that violate $\Delta$-regularity in the root group: In case of $n \cdot \Delta$ being odd, a single terminal node of some root group member may have an unused connection left.

The following Theorem 1 shows that an admissible overlay graph always exists.

**Theorem 1.** For every graph $G$ with $n \geq \Delta^2$, there exists an admissible overlay graph $G'$.

*Proof.* Every admissible topology consists of $A \cdot \Delta + B \cdot (\Delta - 1)$ leaf nodes, where $A = |\text{members}(g_{\text{root}})|$ with $2 \leq A \leq \Delta$ and $B \geq 0$ is the total number of all other subgroups $|\mathbb{G} \setminus \text{members}(g_{\text{root}})|$: Informally, every of the $A$ subgroups of $g_{\text{root}}$ has $\Delta$ nodes, and every of the $B$ other subgroups is attached to the subtree rooted at some member of $g_{\text{root}}$. Attaching is done by replacing one node (thereby reducing the number of leaf nodes by 1) with a group consisting of $\Delta$ new nodes, which amounts to a total of $\Delta - 1$ new nodes.

The formal proof is by induction. For $n_0 = \Delta^2$ the claim follows trivially, since $\Delta^2 = \Delta \cdot \Delta + 0 \cdot (\Delta - 1)$ implies that there is a topology with $\Delta$ members in $g_{\text{root}}$ and no subgroup of a root member. For $n_i = n_{i-1} + 1$, $i \geq 1$, there are two cases:

1. If the number of members in $g_{\text{root}}$ in the topology for $n_{i-1}$ is $2 \leq |\text{members}(g_{\text{root}})| \leq \Delta - 1$, then $n_{i-1} = A \cdot \Delta + B \cdot (\Delta - 1)$ with $B \geq 1$ since $A \leq \Delta$ and $n_{i-1} > \Delta^2$. Consequently, for $n_i$, a topology $n_i = (A + 1) \cdot \Delta + (B - 1) \cdot (\Delta - 1)$ exists. For example, any subgroup $g_i \notin \text{members}(g_{\text{root}})$ can be replaced by the additional node, whereas $g_i$ becomes member of the root group.

2. If the topology for $n_{i-1}$ satisfies $|\text{members}(g_{\text{root}})| = \Delta$, then $n_{i-1} = \Delta \cdot \Delta + B \cdot (\Delta - 1)$, and a topology $n_i = 2 \cdot \Delta + (B + \Delta - 1) \cdot (\Delta - 1)$ exists for $n_i$. For example, we could just remove $\Delta - 2$ groups from the root group and replace some arbitrary node by a group consisting of the $\Delta - 2$ removed groups, the replaced node, and the single additional node.

Hence, the claimed existence of at least one topology for every graph $G$ with $n \geq \Delta^2$ follows. $\square$

By introducing a suitable minimum criterion, we can even stipulate the existence of a unique minimal admissible overlay graph. Although it is not necessarily the global minimum-weight overlay graph, it is nevertheless the minimum one w.r.t. all alternative admissible topology constructions found by our method. Informally, the minimum criterion for choosing group members requires that the selected group has minimal total weight of all internal connections, and that all members agree upon this fact. It may hence be the case that there is a lower-weight choice for some members, but no one that is better for all members of any alternative group.

**Definition 3.** An admissible overlay graph $G'$ is minimal if, for all members $x \in \text{members}(g_i)$ of any group $g_i \in \mathbb{G}$ in the corresponding topology tree, no other group $g'_i$ can be built with $x \in \text{members}(g'_i')$, $\omega(g'_i') < \omega(g_i)$ and $\forall y \in \text{members}(g'_i') : \omega(g'_i') < \omega(g_y)$, where $g_y$ is the (unique) group in the topology tree with $y \in \text{members}(g_y)$.

The minimum admissible graph is well-defined: Choosing a new group according to this criterion does not lead to a violation of the minimality of any already existing group in the final topology, since Definition 1 ensures that a higher-level group has a higher weight than any of its members. The following Theorem 2 proves that the minimum admissible overlay graph exists and is unique.

**Theorem 2.** For every graph $G$ with $n \geq \Delta^2$, the minimal admissible overlay graph $G'$ is unique.

*Proof.* By contradiction. Let us assume that there exist two admissible overlay graphs $G'_1$ and $G'_2$, which are both minimal. Going up the topology tree of $G'_1$ and $G'_2$, at some depth the group structure must be different. More specifically, there must be a group member $x$ in some group $g_i$ in $G'_1$ which is member in some other (not corresponding) group $g_j$ in $G'_2$. Recall that a node or a group can only be member of at most one group. However, either $g_j$ or $g_i$ in $G'_2$ has lower weight, which implies that the admissible overlay graphs $G'_1$ and $G'_2$ cannot both be minimal according to Definition 3. $\square$

The following theorems establish some important properties of our overlay graphs.

**Theorem 3.** Each admissible overlay graph $G'$ is $\Delta$-regular if $n \cdot \Delta$ is even.

*Proof.* Obvious from the topology construction. $\square$

**Theorem 4.** Each admissible graph $G'$ has $\left\lceil\frac{\Delta}{2}\right\rceil$ groups, including the root group, such that $|\mathbb{G} \cup \{g_{\text{root}}\}| = \left\lceil\frac{\Delta}{2}\right\rceil$ different group identifiers are needed in the final tree.
Proof. W.l.o.g., we fill up the root group with $0 \leq m \leq \Delta - 2$ dummy nodes until $g_{\text{root}}$ has $\Delta$ members, which does not change the tree structure below the root group. The resulting topology tree is a $\Delta$-ary tree with $n' = n + m$ leaf-nodes, which has $\left\lceil \frac{n'}{\Delta - 1} \right\rceil$ internal-nodes according to [8, Ex. 2.3.4.5.6]. We prove that $X = \left[ \frac{n'}{\Delta - 1} \right]$ is the only integer number $X \in \mathbb{N}$ for $0 \leq m \leq \Delta - 2$ that equals $\frac{m}{\Delta - 1}$: Since obviously $\left\lceil k - x + 1 \right\rceil > \left\lceil k \right\rceil = k$ for any integer $k \geq 0$ and $0 \leq x < 1$, choosing $k = \frac{m}{\Delta - 1}$ and $x = \frac{m}{\Delta - 1}$ reveals $X + 1 = \left[ \frac{n'}{\Delta - 1} + 1 \right] > \left[ \frac{n'}{\Delta - 1} \right] + \left[ \frac{m}{\Delta - 1} \right] = \frac{n' + m}{\Delta - 1} = \frac{n'}{\Delta - 1}$ as required.

Next we show that our admissible overlay graph is $\Delta$-connected. We first need a few preliminary lemmas for this purpose.

Lemma 1. Let $g$ be a group and $T_g \subseteq \text{nodes}(g)$ with $|T_g| = \Delta$ be its set of terminal nodes. Then, every $p \in \text{nodes}(g), g \in (\mathbb{G} \cup \Pi)$, has $\Delta$-node-disjoint paths to the nodes in $T_g$.

Proof. Induction on depth $d$ in the topology tree. For $d = 0$, $g$ is a node; the claim hence follows trivially from our convention. For some $d > 0$, suppose that the claim holds for every sub-group $g_i \in \text{members}(g)$. Clearly, $p \in \text{nodes}(g_i)$ for some specific $i$ and has hence $\Delta$-node-disjoint paths to $T_{g_i}$. By the construction of $g$, every $q \in T_{g_i} \setminus T_g$ is connected to one terminal node $q_i$ in every $T_{g_i}$ with $g_j \in \text{members}(g) \land g_j \neq g_i$. Applying the induction hypothesis again, $q_i$ is connected to every node in $T_{g_i}$ and hence also to the one in $T_{g_j} \cap T_g$. For $q \in T_{g_i} \cap T_g$, a connection to some node in $T_g$ is obvious.

Lemma 2. Let $S \subseteq T_g$ with $|S| \leq \frac{\Delta}{2}$ be a subset of group $g_i$'s terminal nodes, where $g_i \in \mathbb{G}$. Then, any $p \in S$ is connected to any terminal in $T_{g_i} \setminus S$ via $|S|$ node-disjoint paths.

Proof. From Lemma 1, it follows that any terminal $q \in T_{g_i}$ is connected to every terminal of its subgroup. Hence, w.r.t. connectivity, groups and single nodes are equivalent. Since the subgroups in $g_i$ are fully connected, the lemma follows trivially.

Lemma 3. Let $g_{\text{root}}$ be the root group, consisting of $2 \leq x \leq \Delta$ members and $n \geq \Delta^2$. Then the terminal nodes of any two sub-groups $g_a$ and $g_b \neq g_a$ are connected via $\Delta$ node-disjoint paths in case of $n \cdot \Delta$ is even, or $\Delta - 1$ node-disjoint paths otherwise.

Proof. Each group has $\Delta \cdot (\Delta - 1)$ group internal connections, which is always even. Since the number of groups is an integer, the number of connections in the root group is even if $n \cdot \Delta$ is even or odd if $n \cdot \Delta$ is odd. Therefore a ring connecting $\left\lceil \frac{\Delta}{2} \right\rceil + \left\lceil \frac{\Delta}{2} \right\rceil$ terminal nodes of every subgroup with its neighboring subgroups can be built if $n \cdot \Delta$ is even. The same ring with one connection missing can be built if $n \cdot \Delta$ is odd. The claimed connectivity follows from Lemma 2.

Theorem 5. Each admissible graph $G'$ with $n \geq \Delta^2$ is $\Delta$-node-connected if $n \cdot \Delta$ is even, or $\Delta - 1$-node-connected if $n \cdot \Delta$ is odd.

Proof. Let $p$ and $q$ be two arbitrary nodes and $g \in \mathbb{G}$ be the lowest-level group such that $p \in \text{nodes}(g)$. We first show that $p, q$ are connected via $\Delta$ respectively $\Delta - 1$ node-disjoint paths, which implies the statement of our theorem via Menger’s Theorem [15].

1. If $g_i$ is a regular group, $p \in \text{nodes}(g_a)$ and $q \in \text{nodes}(g_b)$ with $g_a, g_b \in \text{members}(g)$ and $g_a \neq g_b$, we claim that there are $\Delta$ node-disjoint paths between the two sets of terminals $T_{g_a}$ and $T_{g_b}$. By the construction of a group, $\Delta - 1$ pairs of nodes of $T_{g_a} \times T_{g_b}$ are connected by disjoint paths routed over at most one sub-group. According to Lemma 2, there is always a connection between any pair of terminals of this sub-group. Hence, we only have to show that the external connections of $T_{g_a}$ and $T_{g_b}$ are connected outside of $g$ as well. Consider the parent group $g'$ of $g$.

(a) If the terminal nodes $p' \in T_{g_a}$ and $q' \in T_{g_b}$ of group $g$ are non-terminals in $g'$, they are connected in $g'$ via at most one other subgroup of $g'$, according to Lemma 2.

(b) If w.l.o.g. $p' \in T_{g'}$ but $q' \notin T_{g'}$, then $q'$ is connected to any terminal $q^* \in T_{g'}$ via at most one other subgroup of $g'$ by Lemma 2.

(c) If both $p'$ and $q'$ (or $q^*$) are terminals in $g'$, go up to the parent group of $g'$ until either (a) applies or the root group is reached, where Lemma 3 guarantees connectivity between $p'$ and $q'$.

2. $g$ is the root group $g_{\text{root}}$, with $p \in \text{nodes}(g_a)$, $q \in \text{nodes}(g_b)$, $g_a, g_b \in \text{members}(g)$ and $g_a \neq g_b$. The required connectivity follows immediately from Lemma 3.

Applying Menger’s Theorem, the claimed connectivity follows.

The above results reveal several advantages of our approach. First of all, connection weights may be arbitrary; in particular, they need not satisfy the triangle inequality. Moreover, by adding additional constraints to Definition 3, overlay graphs with specific additional properties can be built. For example, in [12], a CDMA wireless network is considered where every node is connected to neighbors at about the same distance. Our topology construction can easily be adapted to favor this situation.
If the weights in the communication graph reflect physical distance, our minimal topology construction clusters nodes according to their spatial density. Nodes that are close to each other will be near the leafs of the topology tree, which leads to a nice failure-locality property of our overlay graph: Catastrophic failures in a spatially localized area will affect communication only in the immediate neighborhood. Combined with additional measures for self-healing, by continuous adaption of the overlay graph, this leads to excellent fault-tolerance properties.

In fact, failures that hit some part of the tree do not severely—if at all—impair fault-tolerant communication in other parts: The proof of Theorem 5 reveals that failures that completely wipe out all members of some group \( g \) do not impair \( \Delta \) node-connectivity of the remaining tree that results from purging the subtree rooted at \( g \). Moreover, all nodes within some intact subtree rooted at some member (or sub-member) of \( g \) are still \( \Delta - 1 \) node-connected with each other, since only the single external connection routed via \( g \) is cut by the failures in \( g \).

### 4 Conclusions

Figure 2 depicts an example overlay graph \( G' \) for \( n = 100 \) and \( \Delta = 3 \). The dotted connections denote the \( g_{\text{root}} \) internal connections.

![Figure 2. Example Overlay Graph](image)

We presented a method for constructing overlay graph and building a hierarchy of clusters in wireless sensor networks and other large-scale distributed systems. The overlay graph is \( \Delta \)-regular, \( \Delta \)-node-connected, ensures failure locality and has low weight.

Future work in this area will focus on topology constructing algorithms and performance evaluations. There are also some novel applications of our algorithm, which require in-depth study.

### References


